Introduction

Using finite-difference modeling for solving differential equation system of elastic vibrations is one of the ways to solve seismic prospecting direct problem.

For this purpose it is necessary to set initial conditions for obtaining unique solution of differential equation which satisfies to require conditions such as source position, impulse type and impulse frequency.

While modeling wavefields it is native to set the initial conditions on sphere (circle) under the features of Lame's equations system fundamental solution. In this work a ray-tracing method of initial conditions definition was used.

In this case, using of ray-tracing method is possible only when initial perturbation enclosed by homogeneous layer limits. Otherwise, it is necessary to calculate a wave that left initial layer and came to neighboring one, but also a wave refracted from boundary, which makes our problem more complicated. Such limitations lead to some problems with setting of initial conditions. The most important task is to minimize wave front aliasing effect caused by calculations on a grid with fixed cell size.

There are two ways to solve this problem.

Here is an illustration of initial perturbation (Ricker's impulse with frequency 40 Hz) derived by the use of ray-tracing method on a circle with radius 50 m, grid step size dx = dz = 4 m and wavefield at 300 m from shot point (*fig. 1*).

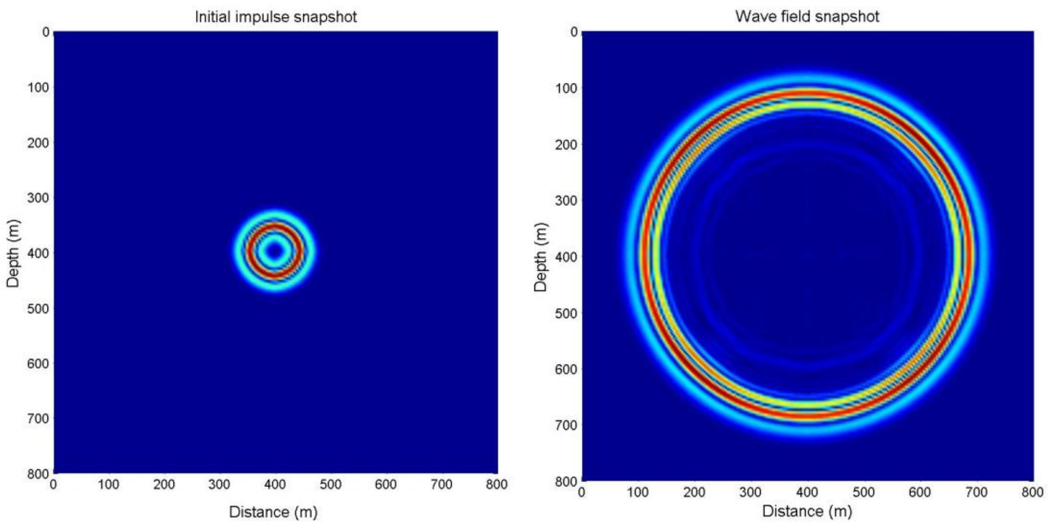

Fig. 1. Initial source definition (**left**) and wavefield at 300 m from shot point (**right**)

Using of regular grid

1. Expanding of the initial radius

Such method of distortions removing while setting initial conditions is the simplest and obvious. By expanding of initial perturbation radius with keeping necessary grid step it is able to get more accurate representation of wave front on the grid. Enlarging the radius one can see the reduction of the aliasing effect of initial wave front.

Though, such way is very limited one due to fixed size of layers. And it makes impossible to expand initial perturbation radius larger then initial layer thickness.

2. Reducing grid step

As an alternative way to improve quality of initial perturbation and consequently further wave pattern is to reduce grid step. It allows to set initial condition on grid more accurate.

Therewith, in further computations, data accumulates fewer errors due to smaller grid size.

Also, this method is designed for solving yet another problem: possibility of reaching homogeneous layer limits while growing of initial sphere radius. Because, in this case it is not necessary to expand layer limits (correction is performed by reducing size grid cells).

However, this method causes a significant increase of wavefield calculation time because of growing number of grid nodes.

Using of irregular grid

The way to modify the described method is to use an irregular grid with confluence region in the range of initial perturbation. It allows to avoid disadvantages of previous methods. Using such a modification make it possible to define initial perturbation more accurately, sphere's radius will not grow. As a result, it is impossible to exceed the bounds of homogeneous layer and there is no significant growth of grid nodes which causes calculation time increase (*fig. 2*).

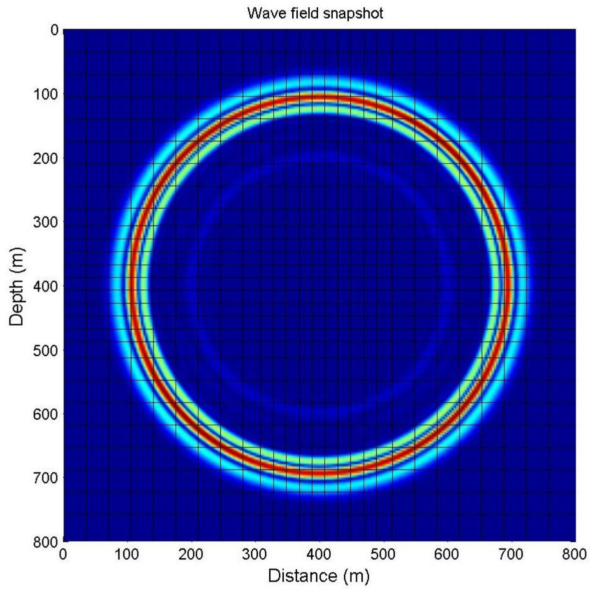

Fig. 2. Wavefield at 300 m from shot point (initial perturbation radius – 100 m, grid steps $dx_1 = dz_1 = 4$ m, $dx_2 = dz_2 = 2$ m, using Ricker's impulse with frequency 40 Hz)

Method of comparison

Comparison of pronounced methods of reducing an initial error is performed as an analysis of correlation between wavefields. Calculation was performed in homogeneous media. The media size was 1600 x 3200 m and P-wave velocity equaled to 2000 m/s. Finite-difference time domain step was 0.1 ms. Wavefield, calculated on a grid with cell size 2 x 2 m and initial perturbation radius 100 m was taken as a base. By setting of such parameters, high accuracy of initial impulse definition and further calculations were carried out, because of big radius of initial perturbation zone and relative small step by the media.

Comparison was made between four wavefields. The first wavefield was calculated with the roughest definition of initial impulse. It was made by setting big grid step by the media – 4 x 4 m and small radius of initial perturbation – 50 m. Other three wavefields were calculated by using the mentioned methods of more accurate initial source definition.

Characteristics of model wavefields to compare:

1. Regular grid 4 x 4 m, radius of initial perturbation – 50 m.
2. Regular grid 4 x 4 m, radius of initial perturbation – 100 m.
3. Regular grid 2 x 2 m, radius of initial perturbation – 50 m.

4. Irregular grid with confluence region, overlapping initial perturbation area per 50 m in each direction with the size 2 x 2 m. Other part of modeling area is calculated on a grid with the step 4 x 4 m, radius of initial perturbation is 50 m.

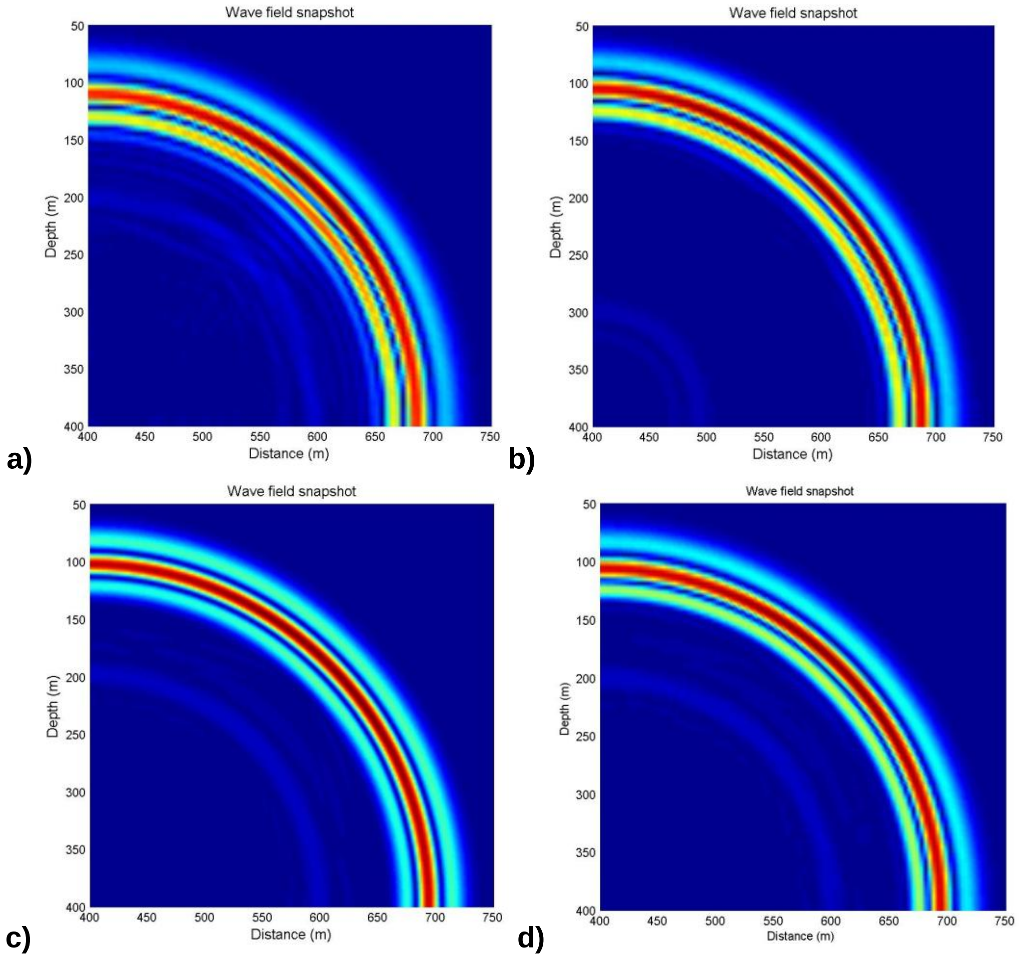

Fig. 3. Fragments of model wavefields at 300 m from shot point:
regular grid, dx = dz = 4 m, radius of initial perturbation – 50 m (**a**),
regular grid, dx = dz = 4 m, radius of initial perturbation – 100 m (**b**),
regular grid, dx = dz = 2 m, radius of initial perturbation – 50 m (**c**),
irregular grid, $dx_1 = dz_1 = 4$ m, $dx_2 = dz_2 = 2$ m, radius of initial perturbation – 50 m (**d**);
(Ricker's impulse with frequency 40 Hz was used for all cases)

Following table represents the results of performed experiments (*tab. 1*).

Wavefield No.	Correlation coefficient (300 m from shot point)	Calculation time for the first 5000 ms (min : sec)
1	0.9104	127 : 16
2	0.9417	126 : 01
3	0.9966	756 : 06
4	0.9848	268 : 33

Tab. 1. Comparative analysis of methods of increasing initial perturbation definition accuracy

Analysis of results

Analysis of data represented on *tab. 1* shows that the most effective way to define initial impulse more accurately is using a small grid step, the radius of initial perturbation can be decreased up to minimal value to enclose initial impulse completely. But using of such a method increases calculation time dramatically. So, when accuracy of initial perturbation definition decreases by less than one percent as compared with model wavefield, calculation time period increases to twelve hours.

Method of initial radius increase appears to be adequate due to calculation time period (it decreases almost six times, when the accuracy goes down by a little bit more than five percents), but this method is limited, as it is impossible to set the bigger radius of impulse definition.

Using irregular grid make it possible to define initial impulse accurately (losses of wavefield calculation accuracy are a little bit more than one percent) and also to save the whole wavefield calculation time, which is less than the model field calculation time almost for three times.

Nonoptimal choice of grid cell size or impulse definition radius causes obtaining of inaccurate results. In this case, the accuracy decreased almost by nine times, whereas calculation time period decreased by six times.

Conclusion

1. Decrease of sphere (circle) radius in finite-difference method arouses defects caused by inaccurate sampling of sphere (circle) at a big grid step.
2. Decrease of a grid step in the whole model helps to minimize defects of sphere (circle) sampling but leads to calculation time increase.
3. Using irregular grid with confluence region in initial perturbation area allows to decrease accuracy defects with a small calculation time decrease.

References

1. Ivo Oprsal and Jiri Zahradnik. Elastic finite-difference method for irregular grids. *Geophysics*, **64**, 240-250.