ВЕКТОРНАЯ МИГРАЦИЯ СЕЙСМИЧЕСКИХ ВОЛНОВЫХ ПОЛЕЙ НА МАТЕРИАЛАХ ВСП

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Vector Migration of VSP Wave-fields

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Summary

In this paper we introduce a seismic migration method based on an optimization approach. 3C VSP seismic records are used as source data. As a result, a field of vector reflection coefficient is recovered. This 3C coefficient represents an image of the media. There are two significant features of the proposed method. First, it uses input seismic traces as a source function in the back propagation problem. This allows us to use initial data recorded only on the well. Second, a strong separation between migration and inversion procedures guarantees an efficiency and stability of the algorithm constructed for a solution of the problem. It is clear that in this approach the seismic field migrated to any vertical line in an object domain can be processed by means of regular VSP techniques. So, reconstruction of the image of the media appears to be an inversion procedure applied to migrated data on every vertical line in an investigated geological section. For dipping horizons any other direction of the reconstructed profile may be applied. 3C surface data can be also migrated in the same way.

Аннотация

В работе предлагается метод миграции, основанный на оптимизационной постановке. В качестве исходных данных используются трехкомпонентные трассы ВСП. В результате формируется изображение среды в виде векторного коэффициента отражения. Метод характеризуется двумя основными чертами. Во-первых, исходное волновое поле рассматривается в задаче продолжения поля с обратным временем в качестве источника. Это позволяет использовать только данные, зарегистрированные на скважине. Во-вторых, четкое разделение этапов миграции и инверсии гарантирует эффективность и устойчивость алгоритма решения задачи. Очевидно, что при таком подходе продолженное поле может быть подвергнуто стандартной обработке данных ВСП на любом вертикальном сечении исследуемой области, а процесс построения изображения среды оказывается процедурой инверсии в каждом из таких сечений. В случае наклонных границ могут быть использованы сечения произвольных направлений. Предложенная процедура также применима и к векторным данным наземной сейсморазведки.

1. Introduction

In this paper we introduce a seismic migration [1] method based on an optimization approach. As input data 3C VSP seismic records are used for a source located at the daytime surface. It is supposed that an *a'priori* reference model has right kinematical characteristics. A pulse form is a given function.

Let us give a preliminary description of proposed approach before a strict mathematical formulation of the problem is made. The first main concept is that a measured wave-field is used not as a boundary condition [2] but as a wave-field source for a reverse time propagation problem. Such an approach theoretically removes initial conditions and aperture failures.

The second concept is an algorithm stages division into wave-field migration and inversion. A wave-field continuation from a well to a media is a linear problem, but it is also incorrect. In spite of an incorrectness of the problem it is possible to propose a stable method of its solution. A section imaging problem, i.e. a continued wave-field inversion, is a nonlinear incorrect problem. The described division of the algorithm is the mostly effective approach because of an essential *a'priori* information about media characteristics is known. Furthermore, with such approach, it is possible to apply standard VSP processing and interpretation techniques to the migrated wave-field. So, the section imaging process can be considered as an inversion on every vertical line located inside current media domain.

Let us consider the wave propagation problem in terms of elastodynamic formulations as follows:

$$\mathbf{u}_{tt} - L\mathbf{u} = \mathbf{f}, \quad \mathbf{r} \in \Omega, \quad t \in [-t_0, T],$$

where $\mathbf{u}(\mathbf{r},t)$ is a displacement vector, L is the Lame's operator in an inhomogeneous media, and $\mathbf{f}(\mathbf{r},t)$ is a source function. An operator L corresponds to an unknown media characterized by an acoustic impedance $\varkappa = \varkappa(\mathbf{k})$, where $\mathbf{k} = \mathbf{k}(\mathbf{r}) = -(2\varkappa)^{-1}\nabla\varkappa$ is a vector reflection coefficient.

Specifying a reference model with an impedance $\varkappa_0$ corresponding to a reflection coefficient $\mathbf{k}_0$ defines an operator L^0 such as

$$\mathbf{u}_{tt} - L^0\mathbf{u} + \hat{\mathbf{R}}\delta\mathbf{k} = \mathbf{f}, \quad \mathbf{r} \in \Omega, \quad t \in [-t_0, T], \tag{1.1}$$

where $\hat{\mathbf{R}}$ is a tensor defined through a stress tensor $\hat{\boldsymbol{\sigma}}(\mathbf{u}^0)$, $\mathbf{u}^0$ is a solution of (1.1) if $\mathbf{k} = \mathbf{k}^0$.

Non-reflecting boundary conditions are applied for an object domain Ω. Initial displacements are supposed to be zero. As a source information for the migration procedure, vectors of displacements on a well $\Omega_1 \subset \Omega$ is taken:

$$\mathbf{u}(\mathbf{r},t) = \tilde{\mathbf{u}}(\mathbf{r},t), \quad \mathbf{r} \in \Omega_1, \quad t \in [0,T].$$

Thus, a final problem is a reconstruction of an image of a media with an acoustic impedance $\varkappa(\mathbf{k})$ and recovering of a vector coefficient $\mathbf{k}(\mathbf{r})$ at each point $\mathbf{r}$ of the domain $\Omega\backslash\Omega_0$.

2. Wave-field migration

Let us determine the migrated wave-field for source data $\tilde{\mathbf{u}}$ from the domain Ω_1 to Ω as a wave-field which yields the best approximation for $\tilde{\mathbf{u}}$ on Ω_1 and satisfies to the Lame's equation in the domain $\Omega\backslash\Omega_1$. This migrated wave-field $\mathbf{u}(\mathbf{r},t)$ is a solution of the following variation problem with constraints:

$$J(\mathbf{u}) = \int_0^T \int_{\Omega_1} \|\mathbf{u} - \tilde{\mathbf{u}}\|^2 \, d\mathbf{r}dt \to \min_{\mathbf{u}}, \text{ subject to } \mathbf{u}_{tt} - L^0\mathbf{u} = 0, \quad \mathbf{r} \in \Omega, \quad t \in [0,T], \tag{2.1}$$

and non-reflecting boundary conditions. This variation problem was solved by means of the Lagrange coefficients method.

A solution of the variation problem yields a value of $\operatorname{grad} J(\mathbf{u})$. It allows us to construct the migrated wave-field using an iteration minimization techniques. In practice, even the first step gives an acceptable approximation for the migrated wave-field. This can be explained by the fact that the first iteration actually includes full information on primary reflections.

To start the minimization procedure it is necessary to specify an initial approximation $\mathbf{u}^0$. We define it as a solution of the following equation

$$\mathbf{u}_{tt} - L^0\mathbf{u} = \chi(\Omega_1)(\mathbf{u} - \tilde{\mathbf{u}}), \quad \mathbf{r} \in \Omega, \quad t \in [0,T], \tag{2.2}$$

which satisfies to zero initial conditions when $t = T$ and non-reflecting boundary conditions.

Formulations (2.1)–(2.2) lead to: $\operatorname{grad} J(\mathbf{u}) = -\mathbf{u}_t^0$. Then, according to a gradient minimization procedure, we have for a step α_0:

$$\mathbf{u}^1 = \mathbf{u}^0 + \alpha_0\mathbf{u}_t^0, \quad \mathbf{r} \in \Omega, \quad t \in [0,T].$$

3. Inversion of the migrated wave-field

Let us define the imaging problem as the following: in the domain Ω to determine a reflection coefficient $\mathbf{k}(\mathbf{r})$, such as corresponding wave-field $\mathbf{u} = \mathbf{u}_\mathbf{k}$ provides for the best approximation for $\tilde{\mathbf{u}}$ on Ω_1 as $t \in [0,T]$. This statement leads to the following minimization problem:

$$\Phi(\mathbf{k}) = \int_0^T \int_{\Omega_1} \|\mathbf{u} - \tilde{\mathbf{u}}\|^2 \, d\mathbf{r}dt, \text{ subject to } \mathbf{u}_{tt} - L\mathbf{u} = 0, \quad \mathbf{r} \in \Omega, \quad t \in [0,T]. \tag{3.1}$$

It is obvious that the solution of this problem produces the same result as the inversion of the migrated wave-field. Function (2.1) defining the migrated wave-field is quadratic, while function (3.1) is of a higher order. So, we would rather deal with the problem specified by (2.1). Note that function (3.1) has the following important property originating from a nature of the investigated problem. If the field $\tilde{\mathbf{u}}$ exactly corresponds to the media, then function (3.1) has only one point of the minimum and $\Phi_{\min} = 0$.

Analyzing the migrated wave-field we can see that it responds to the main characteristics of the reference media. To determine them we define a gradient minimization procedure for function (3.1). Also we use the method of the Lagrange coefficients for a construction of the $\Phi(\mathbf{k})$ gradient. The appropriate Lagrange coefficient appears to satisfy to the same reverse time problem as was defined earlier for the migrated wave-field.

The detailed analysis of the minimization problem allows us to construct $\operatorname{grad}\Phi(\mathbf{k})$. Then we apply the gradient method at point $\mathbf{k}^0$ that yields the following result:

$$\delta\mathbf{k} = \frac{2\alpha_0}{\varkappa_0}\int_0^T \hat{\boldsymbol{\sigma}}(\mathbf{u}^0)\mathbf{u}^0 dt, \quad \text{as } \alpha_0: \quad J(\mathbf{u}^0 + \alpha\mathbf{u}_t^0) \to \min_{\alpha}. \tag{3.2}$$

Formula (3.2) shows that the reflection coefficient is directly defined by the migrated wave-field. Note that for the one-dimensional case formula (3.2) yields:

$$\delta k(x) = c(k^0)\frac{\partial}{\partial x}\int_0^T \left|u^0(x,t)\right|^2 dt, \tag{3.3}$$

where a value of $c(k^0)$ can be efficiently computed.

4. Numerical modeling

The following numerical experiment is performed to test the proposed method. We consider the media with one internal horizontal boundary and generate the synthetic 3C VSP wave-field in a depth range of 10–870 m (fig.1). A source of P-waves is located at the upper boundary of the domain at a distance of 500 m from the vertical well. P-wave velocities in the upper and lower layer are 2000 m/s and 3000 m/s, respectively, S-wave velocities are equal to the half of the P-wave velocities, a density is constant in the whole domain.

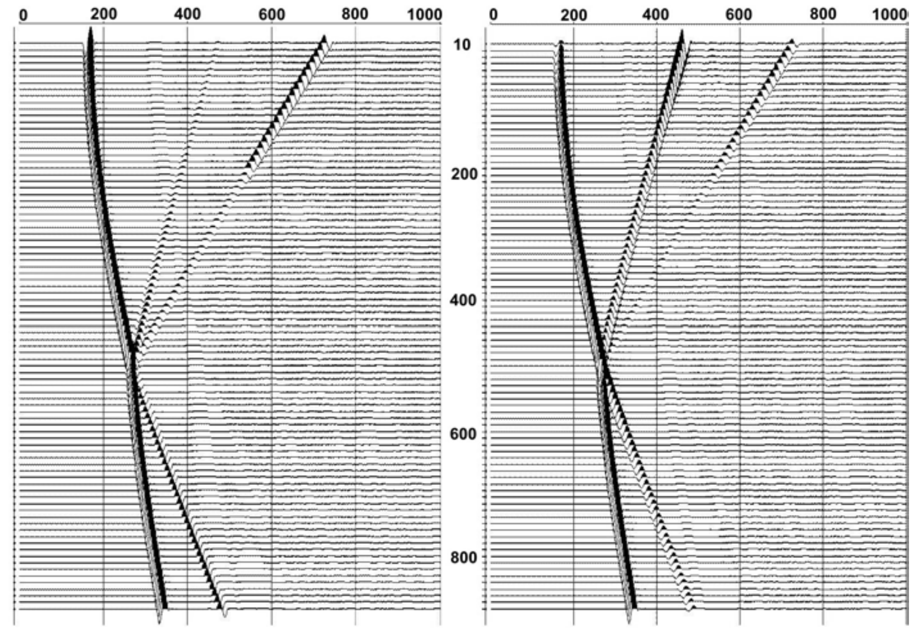

Fig.1. Synthetic wave-field. X-component is on the left, *z*-component is on the right.

For computation of synthetic traces we use a uniform finite-difference scheme for the elastodynamic equation [3]. A spatial grid size for both coordinates is 1 m, a discretization step for time is 0.2 ms. To prevent spurious reflections from the domain boundaries non-reflecting boundary conditions of the second order [4] are used.

The solution of the reverse time problem is measured along a vertical profile located between the shot point and the well at the 100 m distance from the well. The corresponding wave-field is shown in fig.2. The imaging procedure is performed through the simplified algorithm based on formula (3.3). The resulting image of the media presented as a two-component vector reflection coefficient is given in fig.3.

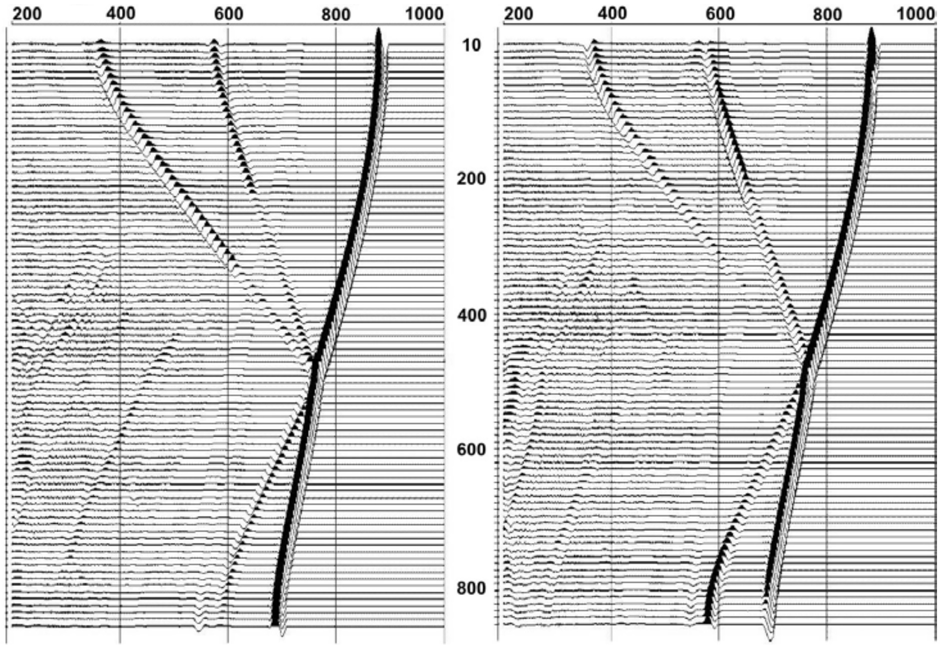

Fig.2. Migrated wave-field (shown in reverse time). X-component is on the left, *z*-component is on the right.

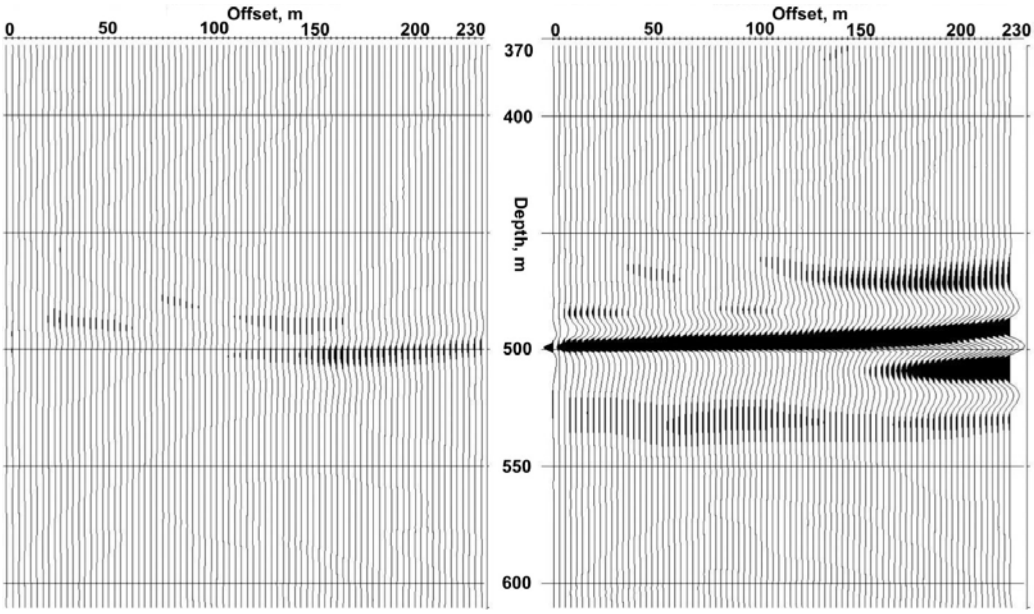

Fig.3. Image of the section. X-component (left) and *z*-component (right) of the reflection coefficient.

5. Conclusion

The migration method for 3C wave-fields is presented in this paper. It provides for a reconstruction of an image of a media, which is actually a field of a vector reflection coefficient. This result is confirmed theoretically and by numerical modeling. Furthermore, a possibility of an imaging by a recovered S-wave reflection coefficient is also supposed. There are no reasons not to apply the same approach to surface 3C seismic data.

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